ECE 333 – Green Electric Energy

3. Energy Conservation Principle

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ENERGY CONSERVATION PRINCIPLE

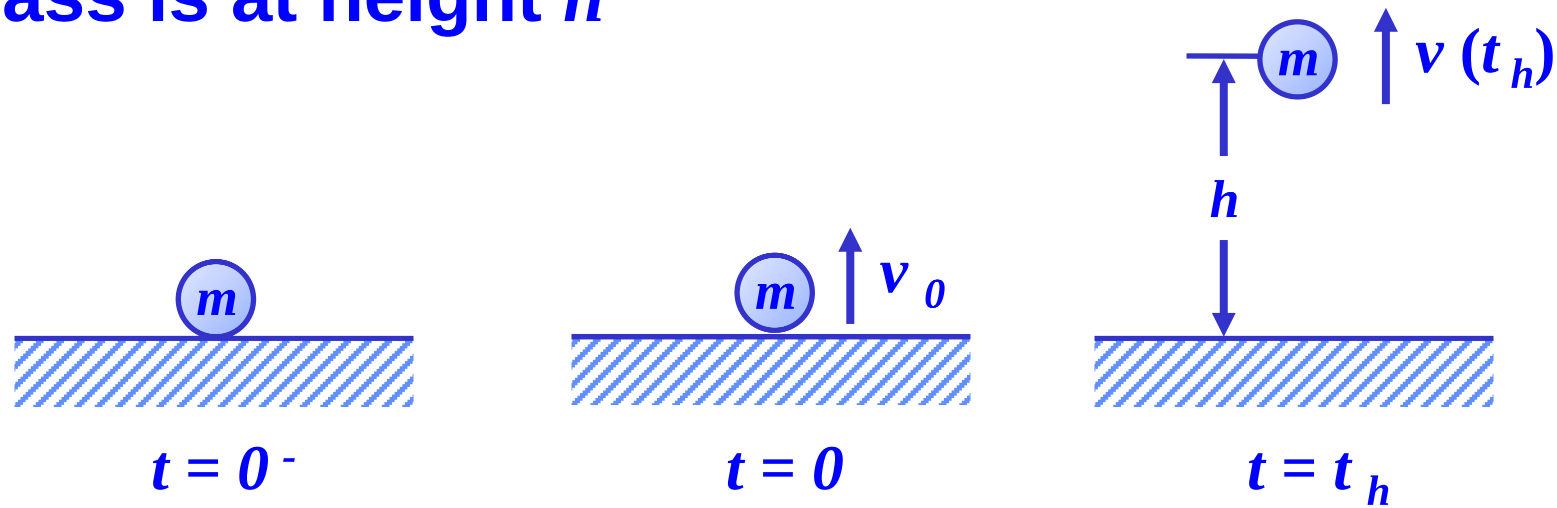
- Energy is, to some extent, an abstract term; for our purposes, we view energy as “work”
- A salient characteristic of energy is its **invariance**: the total energy in the universe remains unchanged over time

ENERGY CONSERVATION PRINCIPLE

- The principle of energy conservation underlies all natural physical, chemical, or biological processes; the principle is, essentially, a very general physical law that owes much to the British physicist Joule
- Indeed, for a purely mechanical system, we can use Newton's laws of motion to derive the energy conservation principle directly

ENERGY CONSERVATION PRINCIPLE

- We examine a very simple example in which a mass m, initially on the ground at time 0^-, is thrown vertically upwards with a speed v_0 at $t = 0$
- We wish to determine the relationship between v_0 and the speed $v(t_h)$ at the time t_h at which the mass is at height h

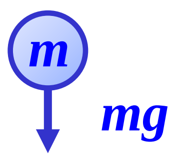


ENERGY CONSERVATION PRINCIPLE

- We use Newton's Second Law to consider the relationship

$$F = ma$$

- Clearly,



$$F = -mg$$

- Let z be the vertical distance traveled by mass m and so its acceleration is $\frac{d^2z}{dt^2}$ and speed is $\frac{dz}{dt}$

ENERGY CONSERVATION PRINCIPLE

- At any instant t,

$$m\frac{d^2z}{dt^2} = F = -mg$$

so that

$$\frac{d^2z}{dt^2} = -g \qquad (*)$$

- The second order differential equation $(*)$ has initial conditions

$$z(0) = 0 \quad and \quad \left.\frac{dz}{dt}\right|_{t=0} = v_0 \qquad (**)$$

ENERGY CONSERVATION PRINCIPLE

- **Since**

$$\frac{dz}{dt} = v(t)$$

then

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{dz}{dt}\right) = \frac{d^2 z}{dt^2} = -g \qquad (**)$$

- **We obtain the solution for $v(t)$ by integrating**

$$\int_0^t \left(\frac{dv}{dt}\right) dt = \int_{v(0)}^{v(t)} dv = v(t) - v(0) = -gt \qquad (***)$$

ENERGY CONSERVATION PRINCIPLE

- **But**

$$v(t) = \frac{dz}{dt} = v_0 - gt$$

and upon integration

$$z(t) - z(0) = v_0 t - \frac{1}{2} g t^2$$

- **At** $t = t_h$ **,** $z(t_k) = h$ **so that**

$$h = v_0 t_h - \frac{1}{2} g [t_h]^2 \qquad (****)$$

and

$$v(t_h) = v_0 - g t_h \qquad (*****)$$

ENERGY CONSERVATION PRINCIPLE

- **From** $(*****)$

$$t_h = \frac{v_0 - v(t_h)}{g}$$

so that $(****)$ **becomes**

$$h = v_0 \frac{v_0 - v(t_h)}{g} - \frac{1}{2} g \bullet \frac{\left[v_0 - v(t_h)\right]\left[v_0 - v(t_h)\right]}{g \bullet g}$$

$$= \frac{v_0 - v(t_h)}{2g} \left\{ 2v_0 - \left[v_0 - v(t_h)\right] \right\}$$

$$= \frac{v_0 - v(t_h)}{2g} \left[v_0 + v(t_h)\right]$$

$$= \frac{1}{2g} \left[v_0^2 - \left[v(t_h)\right]^2 \right]$$

ENERGY CONSERVATION PRINCIPLE

- **We rearrange and obtain**

$$gh = \frac{1}{2}v_0^2 - \frac{1}{2}\left[v\left(t_h\right)\right]^2$$

and multiply by m to get

$$\frac{1}{2}m\left[v\left(t_h\right)\right]^2 = \frac{1}{2}mv_0^2 - mgh \qquad (\dagger)$$

ENERGY CONSERVATION PRINCIPLE

- The relationship can be associated with an energy interpretation since the kinetic energy of mass m at speed $v(t)$ at time t is given by $\frac{1}{2} m \left[v(t) \right]^2$; also, the potential energy of mass m at height h is mgh
- Therefore, we can restate $(\dagger)$ as

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m \left[v(t_h) \right]^2 + mgh \qquad (*\dagger)$$

ENERGY CONSERVATION PRINCIPLE

- In words, at any height *h*

$$\left. kinetic\ energy \right|_{h} + \left. potential\ energy \right|_{h} = \underbrace{\left. kinetic\ energy \right|_{0}}_{constant}$$

and so for two arbitrary values of *h*, say *h'* and *h"*

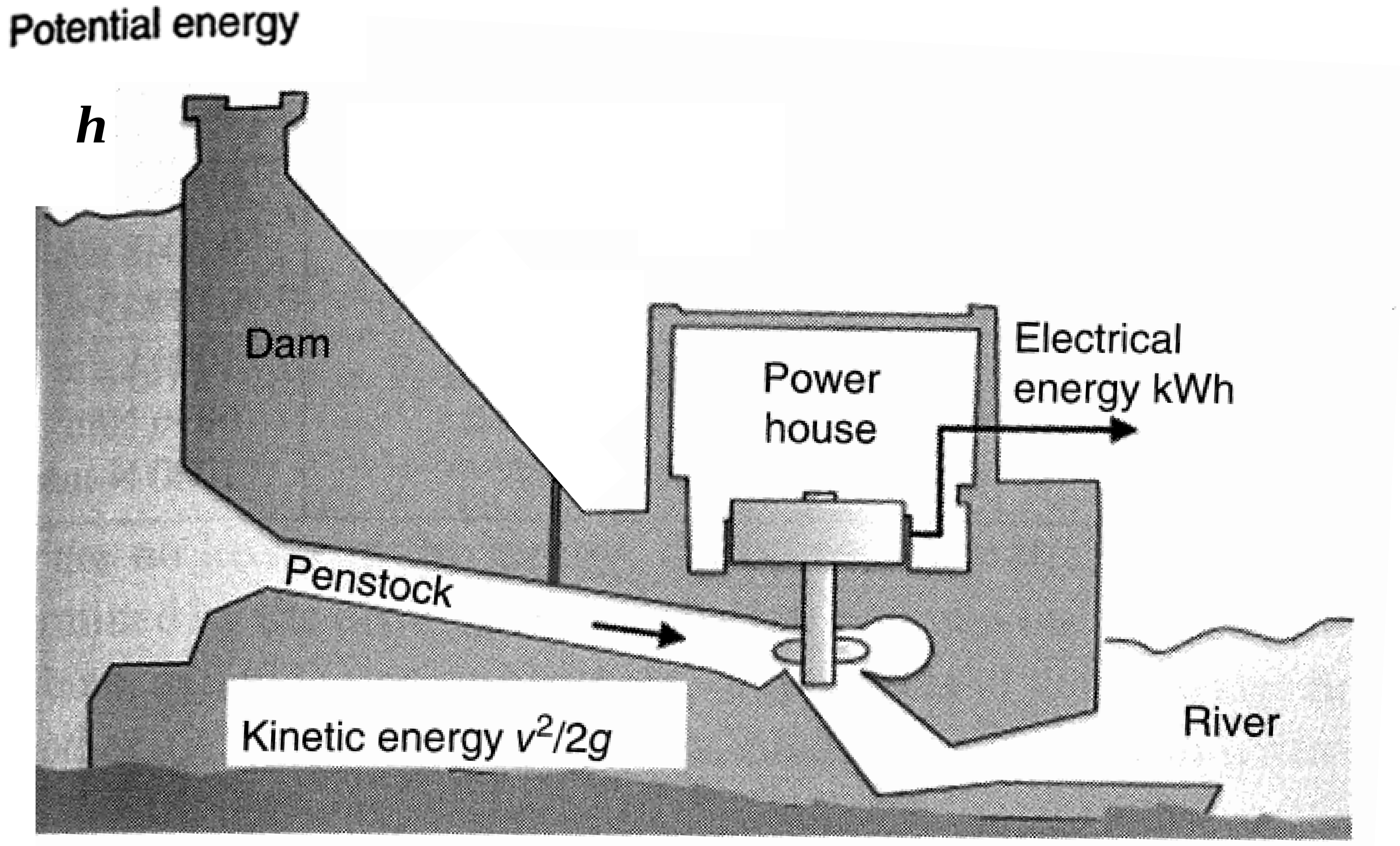
$$\left. kinetic\ energy \right|_{h'} + \left. potential\ energy \right|_{h'} = \left. kinetic\ energy \right|_{h''} + \left. potential\ energy \right|_{h''}$$

- We derived in a straightforward way that total energy of mass *m* is invariant

APPLICATION: ENERGY CONVERSION

- **The energy conservation principle also holds whenever we convert energy from one form into another**
- **We consider a hydroelectric system where a reservoir stores water behind a dam at some height h: the water flows through a penstock and drives a turbine, whose rotor is connected through a mechanical shaft to the electrical generator**

HYDROELECTRIC ENERGY GENERATION



HYDROELECTRIC ENERGY GENERATION

- The water at height h – typically, called the head h – has potential energy, which is converted into mechanical energy as the water flows through the penstock; the mechanical energy drives the turbine and is converted into electric energy by the generator
- Each unit volume of water in the reservoir has mass ρ, where ρ is the density of water, and so has potential energy $\rho g h$

HYDROELECTRIC ENERGY GENERATION

- As each unit traverses the penstock, its potential energy is converted into kinetic energy due to the speed v at which it arrives at the turbine; each unit of volume of water has kinetic energy $\frac{1}{2}\rho v^2$
- We assume that the pressure energy is negligibly small and there are no losses in the system (frictionless penstock) so that the energy conservation law for the mass of the unit volume of fluid results in

HYDROELECTRIC ENERGY GENERATION

$$\rho g h = \frac{1}{2}\rho v^2$$

- The energy conservation law applies to every energy conversion process; however, each process has losses due to inefficiencies of the process and so some of the energy is converted into such losses

HYDROELECTRIC ENERGY GENERATION

- As we shall see, the wind speed air mass has kinetic energy which rotates the wind turbine, which connects to the rotor of an electric generator to convert that kinetic energy into electricity
- Similar notions hold, for example, for a steam generation plant

FOSSIL-FUEL FIRED STEAM GENERATION PLANT